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Effects of tunneling coupling on plasmon modes in asymmetric double-quantum-well structures

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Abstract. The collective charge density excitations in asymmetric double-quantum-well (DQW) structures with different tunneling strengths are systematically studied. In particular, the damping properties of the plasmon modes in various tunneling strengths are investigated in detail. It is shown that plasmon modes in asymmetric DQW structures are quite different from those in symmetric DQW systems. In weak tunneling regime, an intra-subband mode ω_- with an acoustic-like dispersion relation which is damped in symmetric DQW structures arises and coexists with the optical-like mode ω_+ while the inter-subband mode ω_{10} is highly damped. With the tunneling strength being increased, the ω_{10} branch gradually becomes undamped and emerges out of the (1–0) single-particle continuum, whereas the ω_- branch gradually approaches the (0–0) single-particle continuum. In intermediate coupling regime, these three branches of modes coexist undamped. In strong tunneling regime, ω_- enters the (0–0) single-particle continuum and becomes damped. Consequently, only the ω_+ and ω_{10} modes exist in this regime.

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1 Introduction

Due to its importance in revealing the electron-electron many-body interactions such as the exchange-correlation effects in low electronic density and the Coulomb drag effects, plasmon modes in double-quantum-well (DQW) structures have attracted a great deal of theoretical [1–10] and experimental [11–14] interest over the last two decades. However, most of the studies so far are concentrated on plasmons in symmetric DQW structures, whereas the effects of asymmetry are rarely studied. In a symmetric DQW system without tunneling coupling, the optical and acoustic modes were predicted [1,2] and observed [13,14]. These two modes correspond to the in-phase and out-of-phase inter-layer charge density fluctuations, with long wavelength dispersion relations proportional to $q^{1/2}$ and q, respectively. The effects of tunneling on plasmon modes in symmetric DQW structures have also been investigated extensively [7–9]. It is found that the acoustic mode develops a long wavelength gap in the presence of tunneling. It is also pointed out that in symmetric DQW structures the optical and acoustic modes are independent and may cross to each other. In addition to the extensive studies of plasmon modes in symmetric DQW structures, the effects of asymmetry in asymmetric single-quantum-well (SQW) and DQW structures have also been touched upon by several authors. Jain *et al.* studied plasmon modes in an asymmetric SQW and in a DQW structure consisting of two asymmetric SQWs separated by 940 Å [3]. In both structures, the asymmetric nature of the systems results in the coupling and sizable splitting (anti-crossing) of the relevant modes. Recently, plasmon modes in asymmetric double-parabolically graded quantum wells in strong tunneling regime were considered by Wendler *et al.* [9]. Two modes corresponding to the intra- and inter-subband modes in symmetric structures were found. However, different from plasmon modes in symmetric DQW structures, these two modes repulse with each other, leading to the anti-crossing phenomenon similar to Figure 3 in reference [3].

From subband viewpoint, the plasmon modes in symmetric DQW structures can be separated into intra- and inter-subband modes. In the absence of tunneling, the inter-subband mode ω_{10} is just the acoustic branch [7]. In the presence of tunneling, this mode develops a long wavelength gap [8]. In fact, there are two branches of intra-subband modes in symmetric DQW structures, *i.e.*, the ω_+ branch (optical mode) and the ω_- branch. But ω_- is always damped [9]. Hence, only the ω_+ and ω_{10} modes can be observed. In asymmetric DQW structures,

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however, the intra- and inter-subband modes are coupled with each other and can not be separated any more. In view of the qualitative differences between symmetric and asymmetric DQW structures, it is possible that the effects of the spatial asymmetry and mode coupling may make ω_- undamped in certain conditions. In reference [9], the authors only considered the strong tunneling regime and did not find the ω_- mode. If we adjust the many-body interaction in asymmetric DQW structures by controlling the tunneling strength, the originally damped ω_- mode in strong tunneling regime may move out of the single particle continua and the coexistence of the three branches of plasmon modes may be observed.

Motivated by the above mentioned ideas, we systematically study the effects of tunneling on plasmon modes in asymmetric DQW structures. In particular, the variations of the plasmon modes with tunneling strength are investigated in detail. To the best of our knowledge, this is the first systematical investigation in this direction. It is shown that the plasmon modes are affected by tunneling effects significantly, and that an asymmetric DQW structure can support two or three plasmon modes depending on the tunneling strength. In all the cases, the intra-subband mode with long wavelength dispersion $\omega_+ \sim q^{1/2}$ always exists. In weak tunneling regime, the optical-like mode and an intra-subband mode with an acoustic-like long wavelength dispersion relation $\omega_- \sim q$ are supported. Going to the other extreme, we find that an inter-subband mode (ω_{10}) coexists with the ω_+ mode while the originally existing ω_- mode in the weak tunneling regime disappears now in the strong tunneling regime. We show that these three branches can coexist at certain tunneling strength between the weak and strong tunneling regimes. In addition, anti-crossing between the ω_+ and ω_{10} modes can be also seen. It should be noted that in contrast to the symmetric situation, all the modes in asymmetric structures are coupled together. Strictly speaking, there are no pure intra- and inter-subband modes in asymmetric DQW structures. However, we can keep the denotation of the modes in the symmetric case for convenience of discussion.

The rest of this paper is organized as follows. In Section 2, we give the outlined formulas for determining the plasmon modes and a detailed analysis of the effects of tunneling on plasmon modes in asymmetric DQW structures. The numerical results are presented in Section 3. And finally, a brief summary is provided in Section 4.

2 Formulas and analysis

It is well known that the plasmon modes are determined by the zeros of the dynamical dielectric function of the system which can be written as

$$\epsilon_{ll',nn'}(\mathbf{q},\omega) = \delta_{ln'}\delta_{l'n} - U_{ll',nn'}(\mathbf{q})\Pi_{nn'}(\mathbf{q},\omega) \ , \tag{1}$$

where

$$U_{ll',nn'}(\mathbf{q}) = \int\!\!\int \varphi_l^*(z)\varphi_{l'}(z) \times U(\mathbf{q},z,z')\varphi_n^*(z')\varphi_{n'}(z')\,\mathrm{d}z\,\mathrm{d}z' \tag{2}$$

is the Coulomb interaction matrix element and

$$\Pi_{nn'}(\mathbf{q},\omega) = 2\sum_{\mathbf{k}} \frac{f_n(\mathbf{k}) - f_{n'}(\mathbf{k}+\mathbf{q})}{\hbar(\omega + \mathrm{i}0^+) + E_n(\mathbf{k}) - E_{n'}(\mathbf{k}+\mathbf{q})} \tag{3}$$

is the irreducible electron polarizability function. Here

$$U(\mathbf{q},z,z') = \frac{e^2}{2\epsilon_r\epsilon_0 q}\mathrm{e}^{-q|z-z'|} \tag{4}$$

is the two-dimensional Fourier transform of $U(\mathbf{r},\mathbf{r}')$ which stands for the Coulomb interaction between two electrons lying at $\mathbf{r}$ and $\mathbf{r}'$, respectively. For a two-subband model, the plasmon modes are determined by

$$\begin{vmatrix} 1-U_{00,00}\chi_{00} & -U_{00,10}\chi_{10} & -U_{00,11}\chi_{11} \\ -U_{10,00}\chi_{00} & 1-U_{10,10}\chi_{10} & -U_{10,11}\chi_{11} \\ -U_{11,00}\chi_{00} & -U_{11,10}\chi_{10} & 1-U_{11,11}\chi_{11} \end{vmatrix} = 0 \ , \tag{5}$$

where

$$\chi_{nn'}(\mathbf{q},\omega) = \frac{\Pi_{nn'}(\mathbf{q},\omega) + \Pi_{n'n}(\mathbf{q},\omega)}{1+\delta_{nn'}} \ . \tag{6}$$

In symmetric DQW structures, it follows $U_{ll',nn'} = 0$ if $l + l' + n + n'$ is an odd number and thus, equation (5) can be separated into two equations

$$\begin{vmatrix} 1-U_{00,00}\chi_{00} & -U_{00,11}\chi_{11} \\ -U_{11,00}\chi_{00} & 1-U_{11,11}\chi_{11} \end{vmatrix} = 0 \ , \tag{7}$$

and

$$1 - U_{10,10}\chi_{10} = 0 \ , \tag{8}$$

describing the intra-subband mode (ω_+) and the inter-subband mode (ω_{10}), respectively. In the case of zero tunneling, ω_{10} behaves as an acoustic mode in a long wavelength limit, whereas when tunneling effects switch on, it develops a long wavelength gap. Compared with the symmetric counterpart, plasmon modes in an asymmetric DQW structure are quite different. Strictly speaking, $U_{ll',nn'}$ can not be taken as zero when $l + l' + n + n'$ is an odd number. As a result, equation (5) can not be separated into two sub-equations. This means that all the modes are coupled together. However, if the system does not deviate from symmetry seriously, the relationship that the matrix elements with $l = l'$, $n = n'$ are much larger than those with $l+l'+n+n' =$ odd number will be held. In this case, splitting equation (5) into (7) and (8) should be approximately valid.

We first consider the intra-subband modes determined by equation (7). Expanding χ_{00} and χ_{11} in the lowest order of q, we can obtain $\chi_{00}(\mathbf{q},\omega) = N_0 q^2/m^*\omega^2$

and $\chi_{11}(\mathbf{q},\omega) = N_1 q^2/m^*\omega^2$, where N_0 (N_1) is the two-dimensional electronic density of the E_0 (E_1) level. Use these two expressions in equation (7), we can obtain two solutions

$$\omega_\pm^2 = \frac{qe^2}{4m^*\epsilon_0\epsilon_r}\left(f_{00,00}N_0 + f_{11,11}N_1\right) \times \left(1 \pm \sqrt{1 - \frac{4\left(f_{00,00}f_{11,11} - f_{00,11}^2\right)N_0N_1}{\left(f_{00,00}N_0 + f_{11,11}N_1\right)^2}}\right), \tag{9}$$

where

$$f_{ll',nn'}(q) = \iint \varphi_l^*(z)\varphi_{l'}(z)\mathrm{e}^{-q|z-z'|}\varphi_n^*(z')\varphi_{n'}(z')\,\mathrm{d}z\,\mathrm{d}z'. \tag{10}$$

Since $(f_{00,00}f_{11,11} - f_{00,11}^2)N_0N_1 \ll (f_{00,00}N_0 + f_{11,11}N_1)^2$ is satisfied, these two solutions can be approximated as

$$\omega_+^2 \approx \frac{qe^2}{2m^*\epsilon_0\epsilon_r}\left(f_{00,00}N_0 + f_{11,11}N_1\right)\ , \tag{11}$$

and

$$\omega_-^2 \approx \frac{qe^2}{2m^*\epsilon_0\epsilon_r}\left(\frac{N_0N_1}{f_{00,00}N_0 + f_{11,11}N_1}\right)G(q)\ , \tag{12}$$

where $G(q) = f_{00,00}(q)f_{11,11}(q) - f_{00,11}^2(q)$. For symmetric DQW structures, $f_{00,00}(q) \approx f_{11,11}(q) \approx f_{00,11}(q)$ is valid in the long wavelength limit for the whole tunneling. As a result, $\omega_- \to 0$ is valid, which means that this mode lies in the single-particle continua and is highly damped, and therefore, ω_+ is the only undamped intra-subband mode. In asymmetric DQW structures, however, things are quite different. Similar to the symmetric case, the ω_+ branch exists in all the tunneling strengths with an optical long wavelength dispersion relation, *i.e.*, $\omega_+(q \to 0) \sim q^{1/2}$. As for the ω_- mode, we find that it may or may not damped, depending on the tunneling strength. According to our numerical results, $G(q)$ is approximately proportional to q in the long wavelength limit, as shown in Figure 1, where $G(q)$–q relation in asymmetric DQW structures with $w_1 = 22.5$ nm, $w_2 = 20.0$ nm, $V_b = 180$ meV, and $n_{2\mathrm{DEG}} = N_0 + N_1 = 1.6 \times 10^{11}$ cm^{-2} for different barrier widths is displayed. This relationship in conjunction with $f_{00,00}(q) \approx f_{11,11}(q) \to 1$ when $q \to 0$ yields an acoustic dispersion relation for the ω_- mode. From Figure 1, we can also see that the smaller the barrier width b, the smaller the value of $G(q)$ is. This is caused by the fact that the difference between $f_{00,00}$ (or $f_{11,11}$) and $f_{00,11}$ decreases with the increase of the tunneling strength. As a result, the energy of the ω_- mode decreases when the tunneling strength becomes stronger. Thus, the ω_- mode is undamped in the weak tunneling regime but draws closer to the (0–0) single-particle continuum as the tunneling strength increases. Once the tunneling exceeds a critical value, ω_- will be squeezed into the single-particle continua and becomes damped.

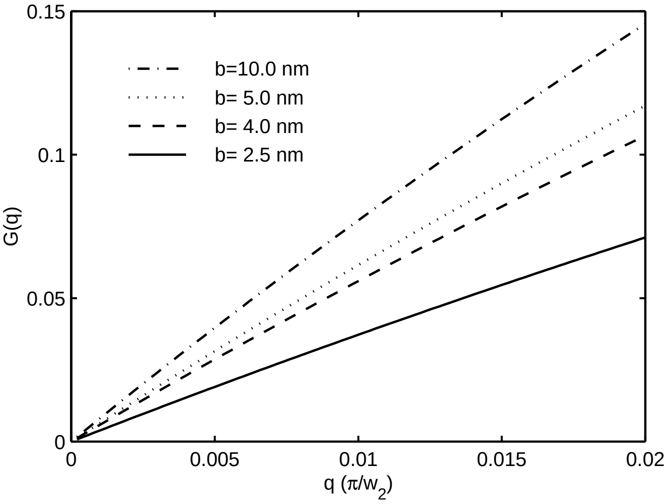

Fig. 1. Calculated $G(q)$–q relation in asymmetric DQW structures with $w_1 = 22.5$ nm, $w_2 = 20.0$ nm, $V_b = 180$ meV, and $n_{2\mathrm{DEG}} = 1.6 \times 10^{11}$ cm^{-2} for $b = 10.0$, 5.0, 4.0 and 2.5 nm. Here w_1 and w_2 are the widths of the two wells, b and V_b are the width and height of the inter-well barrier, and $n_{2\mathrm{DEG}}$ is total two-dimensional electronic density.

We now turn to equation (8) which gives the inter-subband mode ω_{10}. In symmetric DQW structures, ω_{10} lies out of the single-particle continua for all the tunneling strengths. In the finite tunneling regime, ω_{10} has a long wavelength gap. Due to the dynamical many-body effects, it is above the corresponding (1–0) single-particle continuum in the long wavelength limit. With the decrease of the tunneling strength, both ω_{10} and the (1–0) single-particle continuum move downward. In this process, ω_{10} is always above the (1–0) single-particle continuum. Thus, ω_{10} never becomes damped in long wavelength limit in symmetric DQW systems. In asymmetric DQW structures, however, this feature of the ω_{10} mode may be drastically changed by the effects of asymmetry, *i.e.*, ω_{10} may become damped at certain conditions.

3 Numerical results and discussion

To confirm the above mentioned analysis on ω_+, ω_- and ω_{10}, we perform exactly numerical calculations in terms of equation (5) for plasmon modes in asymmetric DQW structures with $w_1 = 22.5$ and $w_2 = 20.0$ nm for four barrier widths: (a) $b = 10.0$, (b) $b = 5.0$, (c) $b = 4.0$ and (d) $b = 2.5$ nm. Here w_1 and w_2 are the widths of the two wells, respectively. In our calculations, the barrier height V_b, defined as the energy difference of the conduction band between the barrier and well layers, is taken as 180 meV, the total two-dimensional electronic density $n_{2\mathrm{DEG}}$ is chosen to be 1.6×10^{11} cm^{-2}, and the background dielectric constant is assumed to be $\epsilon_r = 12.87$ which is appropriate for GaAs. The numerical results are shown in Figure 2. It is seen that the optical-like intra-subband mode (ω_+) exists in all the cases, whereas the acoustic-like intra-subband mode (ω_-) and the inter-subband mode (ω_{10})

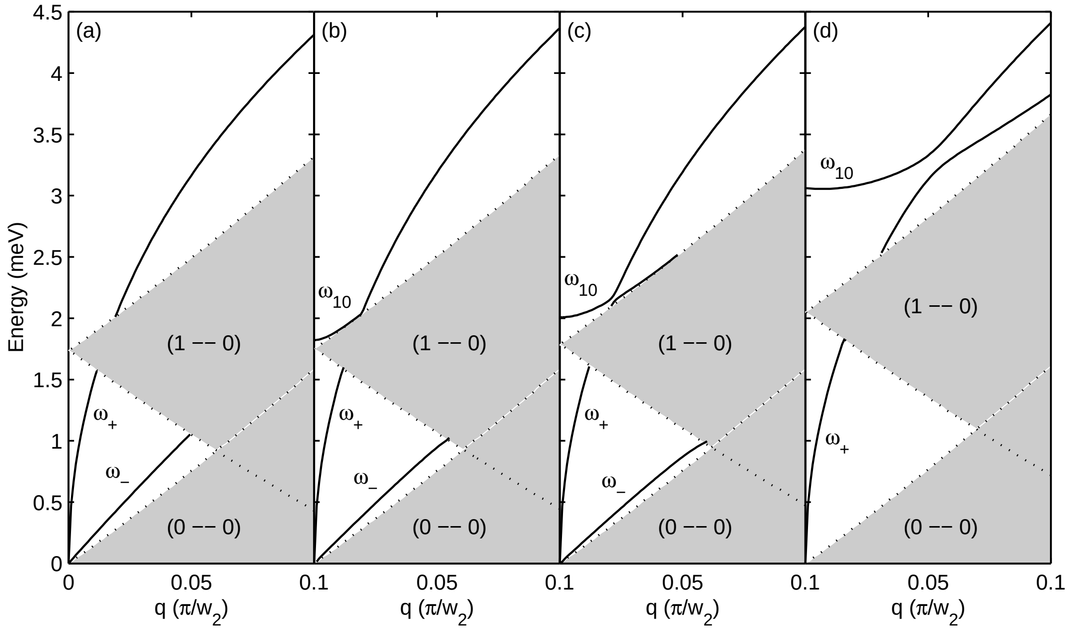

Fig. 2. Dispersion relations of plasmon modes in asymmetric DQW structures with $w_1 = 22.5$ nm, $w_2 = 20.0$ nm, and $V_b = 180$ meV for different barrier widths: (a) $b = 10.0$ nm, (b) $b = 5.0$ nm, (c) $b = 4.0$ nm, and (d) $b = 2.5$ nm. The shaded regions correspond to the single-particle continua.

are damped in the strong and weak tunneling regimes, respectively. In the intermediate tunneling regime, these three modes can coexist. In panels (a, b) and (c), the evolution of the ω_- mode is clearly shown. In weak tunneling regime, ω_- lies above the (0–0) single-particle continuum. With the decrease of b, it moves gradually to and finally enters the (0–0) single-particle continuum and becomes damped. Similarly, as b increases, the gradually approaching of the ω_{10} mode to the (1–0) single-particle continuum can be also clearly seen from panels (b, c) and (d).

Our numerical results show that the effects of spatial asymmetry on the plasmon modes in quantum well structures are essential. These effects stem from the coupling between the inter- and intra-subband modes in asymmetric systems. One phenomenon caused by this mode coupling is the splitting (anti-crossing) of the relevant modes, as has been discussed in references [3] and [9]. A more important effect of the mode coupling is on the damping property of the ω_- and ω_{10} modes in different tunneling strengths. As the central point of our work, the latter is the first systematical investigation in this direction. Wendler *et al.* did not find the ω_- mode because they confined their discussion to the strong tunneling regime in which ω_- is highly damped. Their result corresponds to panel (d) of Figure 2 in our paper. In reference [3], the authors studied the plasmon modes in an asymmetric SQW where the asymmetry was simulated by elevating one-half of the quantum well by a small amount of energy. In fact, this structure can be regarded as a DQW system with a zero width barrier. Bearing this point in mind, we can reasonably understand their numerical result which is just like panel (d) of Figure 2. The authors also presented the plasmon dispersions in a system composed of two asymmetric SQW structures separated by 940 Å (see Fig. 4 in Ref. [3]). Similarly, this system can not be regarded as a DQW structure any more. Instead, it actually should be looked on as a system composed of four quantum wells.

Comparing the plasmon modes in different tunneling strengths, we can find that tunneling has relatively little influence on ω_+, whereas its effects on ω_- and ω_{10} are profound. These effects are embodied in two aspects. On one hand, tunneling effects change the frequencies of these two modes. More importantly, tunneling effects bring essential changes in the damping properties of ω_- and ω_{10}. Since the single-particle continua (0–0) and (1–0) vary little, the changes in the frequencies and damping properties of these two modes are mainly caused by the dynamical many-body effects. It is well known that the depolarization shift of the inter-subband mode, defined as the frequency difference between the collective and corresponding single-particle excitations, measures the magnitude of the dynamical many-body effects. To show the many-body effects clearly, we now turn to the analysis of the depolarization shift of ω_{10} ($\Delta_{10}^{\text{shift}}$). From Figure 2, we can see that $\Delta_{10}^{\text{shift}}$ decreases from 1.01 meV when $b = 2.5$ nm to 0.07 meV when $b = 5.0$ nm, revealing the important effects of tunneling on the dynamical many-body interactions. This implies that we can control the many-body electron-electron interactions by adjusting the tunneling strength. Hence, the evolution of the plasmon modes in asymmetric DQW structures with different tunneling strengths provides a useful tool for studying many-body effects.

As has been pointed out, a new and significant finding of the current work is the existence of a coupling where the acoustic and optical intra-subband plasmon modes and an inter-subband mode coexist undamped. Experimentally, the observation of these three modes simultaneously requires careful control of the consisting well and barrier layers and of the electronic density. Rapid advances in fabrication techniques such as molecular-beam epitaxy and modulation doping methods allow for the manufacture of qualified samples. In addition, inelastic light scattering has become a powerful method to probe the plasmon dispersions in low dimensional systems. For example, the dispersion of the optical and acoustic plasmons in DQW structures was successfully measured using this technique very recently [13,14]. Therefore, it is expected that our prediction can be verified in inelastic light scattering experiment.

4 Summary

In conclusion, we have investigated the effects of the tunneling strength on plasmon modes in asymmetric DQW structures and reported the existence of an undamped intra-subband mode ω_- in the weak and intermediate tunneling regimes which is always Landau damped in symmetric DQW structures. This mode has an acoustic-like dispersion relation in the long wavelength limit. We have also found that the inter-subband mode ω_{10} is Landau damped in the weak tunneling regime which is different from the symmetric case where it never becomes damped. We have shown that these three branches can coexist in certain tunneling strength between the weak and strong tunneling regimes. According to our numerical results, the frequency and the depolarization shift of the ω_{10} mode change remarkably with the variation of the tunneling strength. It can be expected that the characteristics of the plasmon modes in asymmetric DQW systems with different tunneling coupling strengths should provide a useful tool for studying many-body effects.

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